MENIIT

NEET | IIT-JEE | FOUNDATION

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JEE MAIN-2020

COMPUTER BASED TEST (CBT)

DATE : 06-09-2020 (SHIFT-1) | TIME : (9.00 am to 12.00 pm)

Duration 3 Hours | Max. Marks : 300

QUESTION & SOLUTIONS

PART-A : PHYSICS

SECTION – 1 : (Maximum Marks : 80)

Single Choice Type

This section contains 20 Single choice questions. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which Only One is correct.

Full Marks : +4 If ONLY the correct option is chosen.

Negative Marks : –1 (minus one) mark will be deducted for indicating incorrect response.

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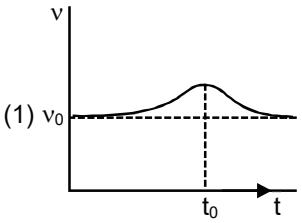
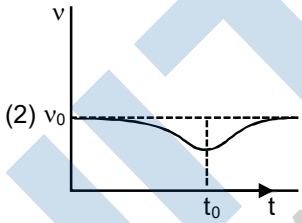
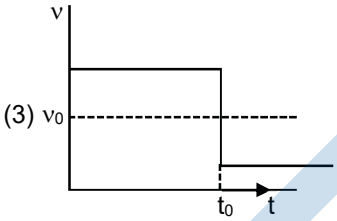
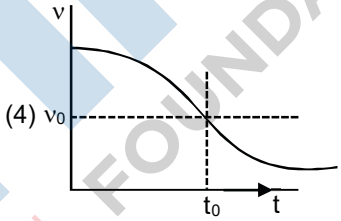
1. A sound source S is moving along a straight track with speed v, and is emitting sound of frequency ν_0. An observer is standing at a finite distance, at the point O, from the track. The time variation of frequency heard by observer is best represented by :

(t_0 represents the instant when the distance between the source and observer is minimum)

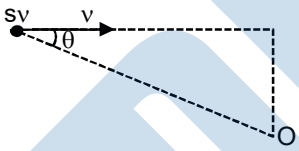
(1) 

(2) 

(3) 

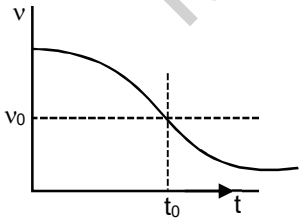
(4) 

Ans. (4)

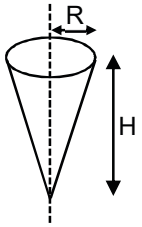
Sol. 

Observed frequency will be $\nu = \frac{v_{sound} + v\cos\theta}{v_{sound}}\nu_0$

Using this formula we can say that observed frequency will decrease continuously and it becomes equal to original frequency when $\theta = 90°$. So correct graph is



2. Shown in the figure is a hollow ice-cream cone (it is open at top). If its mass is M, radius of its top is R and height, H, then its moment of inertia about its axis is

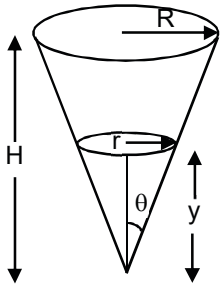


(1) $\frac{M(R^2+H^2)}{4}$ (2) $\frac{MR^2}{3}$ (3) $\frac{MH^2}{3}$ (4) $\frac{MR^2}{2}$

Ans. (4)

Sol. It can be assumed as several parts of discs having different radius, so

$$I=\int dI=\int dm(r^2) \quad \ldots(i)$$



From diagram

$$\frac{r}{y}=\tan\theta=\frac{R}{H}$$

$$r=\frac{R}{H}.y \quad \ldots(ii)$$

$dm = \rho(\pi r^2)dy$ …(iii)

From equation (i), (ii) and (iii)

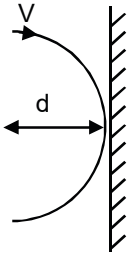
$$I=\frac{MR^2}{2}$$

3. A particle of charge q and mass m is moving with a velocity $-v\hat{i}(v\neq 0)$ towards a large screen placed in the Y-Z plane at distance d. If there is magnetic field $\vec{B}=B_0\hat{k}$, the minimum value of v for which the particle will not hit the screen is :

(1) $\frac{2qdB_0}{m}$ (2) $\frac{qdB_0}{3m}$ (3) $\frac{qdB_0}{m}$ (4) $\frac{qdB_0}{2m}$

Ans. (3)

Sol.

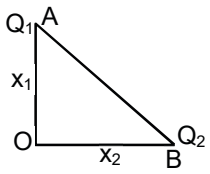


The radius of circular path in given magnetic field is

$$r = d = \frac{mv}{qB_0}$$

$$SO \qquad v = \frac{qB_0 d}{m}$$

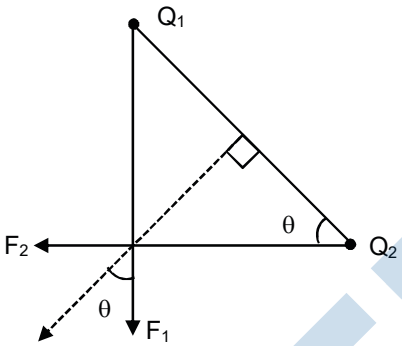
4. Charges Q_1 and Q_2 are at points A and B of a right angle triangle O AB. The resultant electric field at point O is perpendicular to the hypotenuse, then Q_1/Q_2 is proportional to :



(1) $\frac{x_2^2}{x_1^2}$ (2) $\frac{x_1^3}{x_2^3}$ (3) $\frac{x_2}{x_1}$ (4) $\frac{x_1}{x_2}$

Ans. (4)

Sol.



$$\tan\theta = \frac{x_1}{x_2} = \frac{F_2}{F_1} = \frac{KQ_2/x_2^2}{KQ_1/x_1^2}$$

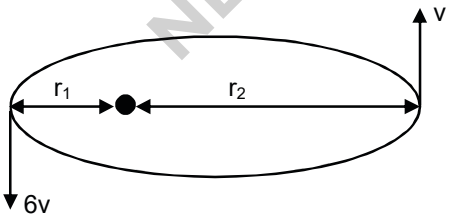
$$\frac{Q_1}{Q_2} = \frac{x_1}{x_2}$$

5. A satellite is in an elliptical orbit around a planet P. It is observed that the velocity of the satellite when it is farthest from the planet is 6 times less than that when it is closest to the planet. The ratio of distances between the satellite and the planet at closest and farthest points is :

(1) 1 : 2 (2) 1 : 6 (3) 3 : 4 (4) 1 : 3

Ans. (2)

Sol.



From angular momentum conservation

$mv_1r_1 = mv_2r_2$

$$\frac{r_1}{r_2}=\frac{v_2}{v_1}=\frac{1}{6}$$

6. An AC circuit has R = 100Ω, C = 2μF and L = 80 mH, connected in series. The quality factor of the circuit is :

(1) 400 (2) 0.5 (3) 2 (4) 20

Ans. (3)

Sol. The quality factor of LCR circuit is given by $Q=\frac{\omega L}{R}=\frac{1}{R}\sqrt{\frac{L}{C}}=\frac{1}{100}\sqrt{\frac{80\times10^{-3}}{2\times10^{-6}}}=2$

7. A point like object is placed at distance of 1 m in front of a convex lens of focal length 0.5 m. A plane mirror is placed at a distance of 2 m behind the lens. The position and nature of the image formed by the system is :

(1) 2.6 m from the mirror, real (2) 1 m from the mirror, virtual

(3) 1 m from the mirror, real (4) 2.6 m from the mirror, virtual

Ans. (4)

Sol. For I_1 applying lens formula

$$\frac{1}{v_1}=\frac{1}{-1}=\frac{1}{0.5}$$

$v_1 = 1$

For I_2 applying condition of plane mirror

for final image (I_3) again applying lens formula

$$\frac{1}{v_3}=\frac{1}{-3}=\frac{1}{0.5}$$

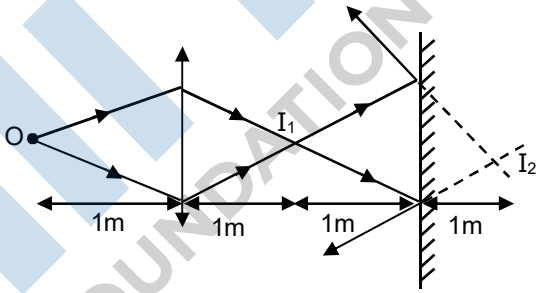
$$v_2=\frac{3}{5}$$

The distance of final image from mirror

$$D=\frac{3}{5}+3$$

D = 2.6

The image will be real (but it is given virtual in answer)



8. An object of mass m is suspended at the end of a massless wire of length L and area of cross-section, A. Young modulus of the material of the wire is Y. If the mass is pulled down slightly its frequency of oscillation along the vertical direction is :

(1) $f=\frac{1}{2\pi}\sqrt{\frac{mL}{YA}}$ (2) $f=\frac{1}{2\pi}\sqrt{\frac{YL}{mA}}$ (3) $f=\frac{1}{2\pi}\sqrt{\frac{mA}{YL}}$ (4) $f=\frac{1}{2\pi}\sqrt{\frac{YA}{mL}}$

Ans. (4)

Sol. $f=\frac{1}{2\pi}\sqrt{\frac{k}{m}}=\frac{1}{2\pi}\left(\sqrt{\frac{YA}{mL}}\right)$

9. If the potential energy between two molecules is given by $U = \frac{A}{r^6} + \frac{B}{r^{12}}$, then at equilibrium, separation between molecules, and the potential energy are :

(1) $\left(\frac{B}{A}\right)^{\frac{1}{6}}, 0$ (2) $\left(\frac{2B}{A}\right)^{\frac{1}{6}}, \frac{A^2}{4B}$ (3) $\left(\frac{2B}{A}\right)^{\frac{1}{6}}, \frac{A^2}{2B}$ (4) $\left(\frac{B}{2A}\right)^{\frac{1}{6}}, -\frac{A^2}{2B}$

Ans. (2)

Sol. $U = \frac{A}{r^6} + \frac{B}{r^{12}}$

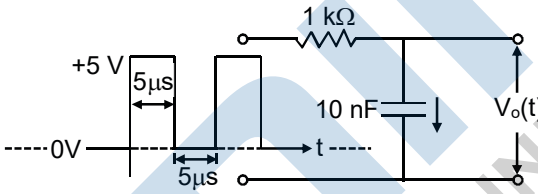
$f = -\frac{dU}{dr}$

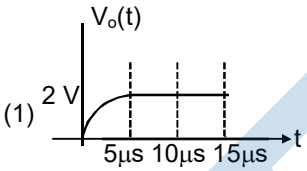
For equilibrium

f = 0

So $r = \left(\frac{2B}{A}\right)^{\frac{1}{6}}, \frac{A^2}{4B}$

10. For the given input voltage waveform Vin(t), the output voltage waveform $V_0(t)$, across the capacitor is correctly depicted by :



(1)


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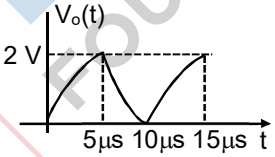

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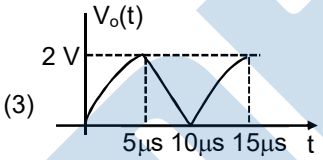
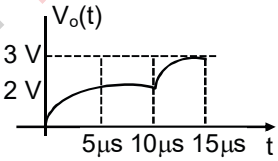


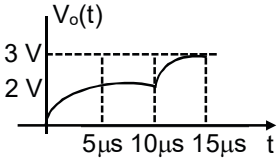
(4)


Ans. (4)

Sol. When first pulse is applied the potential across capacitor is given by $V = V_0\left(1 - e^{\frac{1}{RC}}\right)$

When no pulse is applied, capacitor will discharge like $V = V_0' e^{t/RC}$ when again, second pulse is applied $V = V_0\left(1 - e^{\frac{1}{RC}}\right)$

Using all three equations the correct graph is :



11. A clock has a continuously moving second's hand of 0.1 m length. The average acceleration of the tip of the hand (in units of ms^{-2}) is of the order of :

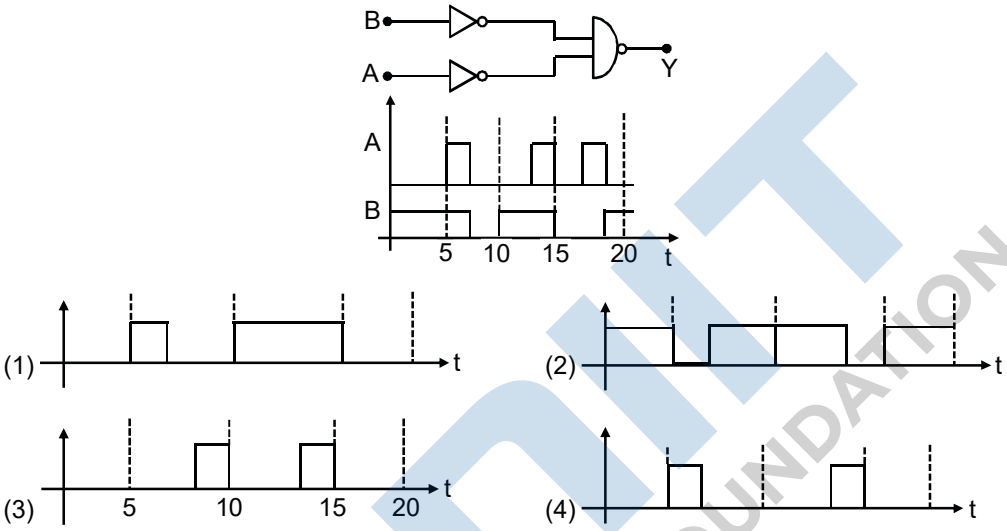
(1) 10^{-3} (2) 10^{-1} (3) 10^{-4} (4) 10^{-2}

Ans. (1)

Sol. The instantaneous acceleration of tip is given by

$$a_c = \omega^2 R = \left(\frac{2\pi}{60}\right) \times 0.1 = \frac{1}{900} \times 10^{-3}$$

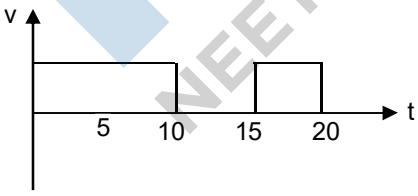
12. Identify the correct output signal Y in the given combination of gates (as shown n) for the given inputs A and B.



Ans. (4)

Sol. Truth table

A	B	Y
0	1	1
1	0	1
0	0	0
1	1	1



13. An electron, a doubly ionized helium ion (He^{++}) and proton are having the same kinetic energy. The relation between their respective de-Broglie wavelength λ_e' λ_{He}^{++} and λ_p is :

(1) $\lambda_e < \lambda_p < \lambda_{He}^{++}$ (2) $\lambda_e > \lambda_{He}^{++} > \lambda_p$ (3) $\lambda_e < \lambda_{He}^{++} = \lambda_p$ (4) $\lambda_e > \lambda_p > \lambda_{He}^{++}$

Ans. (4)

Sol. $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mk}}$

$$\lambda_e : \lambda_p : \lambda_{He^{++}} = \frac{1}{\sqrt{m_e}} : \frac{1}{\sqrt{m_p}} : \frac{1}{\sqrt{He^{++}}}$$

$$\frac{1}{\sqrt{9.1\times10^{-31}}} : \frac{1}{\sqrt{1.6\times10^{-27}}} : \frac{1}{\sqrt{4\times1.6\times10^{-27}}} = 84 : 2 : 1$$

14. An insect is at the bottom of a hemispherical ditch of radius 1 m. It crawls up the ditch but starts slipping after it is at height h from the bottom. If the coefficient of friction between the ground and the insect is 0.75, then h is : ($g = 10\ ms^{-2}$)

(1) 0.20 m (2) 0.60 m (3) 0.80 m (4) 0.45 m

Ans. (1)

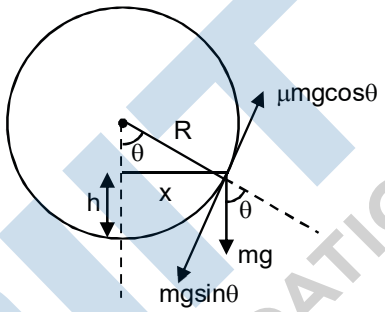
Sol. $mg\sin\theta = \mu mg\cos\theta$

$\tan\theta = \mu = 0.75$

$\Rightarrow \theta = 37$

$$\cos\theta = \frac{R-h}{R}$$

$$\cos 37 = \frac{1-h}{1} \Rightarrow h = 0.2$$



15. You are given that Mass of $^{7}_{3}Li = 7.0160$ u, Mass of $^{4}_{2}He = 4.0026$ u and Mass of $^{1}_{1}He = 1.0079$ u When 20 g of $^{7}_{3}Li$ is converted into $^{4}_{2}He$ by proton capture, the energy liberated, (in kWh), is :

[Mass of nucleon = 1 GeV/c^2]

(1) 1.33×10^6 (2) 6.82×10^5 (3) 8×10^6 (4) 4.5×10^5

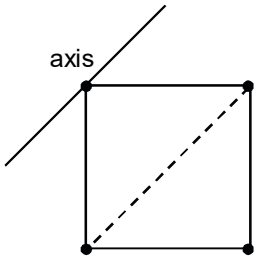
Ans. (1)

Sol. $E = \Delta mC^2$

$E = (1.0079 + 7.0160 - 2(4.0026) \times 931$

$= 1.33 \times 106$

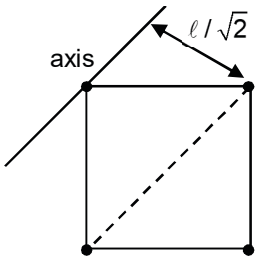
16. Four point masses, each of mass m, are fixed at the corners of a square of side l. The square is rotating with angular frequency ω, about an axis passing through one of the corners of the square and parallel to tis diagonal, as shown in the figure. The angular momentum of the square about the axis is



(1) $m\ell^2\omega$ (2) $3\ m\ell^2\omega$ (3) $4\ m\ell^2\omega$ (4) $2\ m\ell^2\omega$

Ans. (2)

Sol.



$$I = M\left(\frac{\ell}{\sqrt{2}}\right)^2 + m(\sqrt{2}\ell)^2$$

$= 3m\ell^2$

17. Molecules of an ideal gas are known to have three translational degrees of freedom. The gas is maintained at a temperature of T. The total internal energy, U of a mole of this gas, and the value of $\gamma = \left(\frac{C_p}{C_v}\right)$ are given, respectively, by :

(1) $U = \frac{5}{2}RT$ and $\gamma = \frac{6}{5}$

(2) $U = 5RT$ and $\gamma = \frac{7}{5}$

(3) $U = 5RT$ and $\gamma = \frac{6}{5}$

(4) $U = \frac{5}{2}RT$ and $\gamma = \frac{7}{5}$

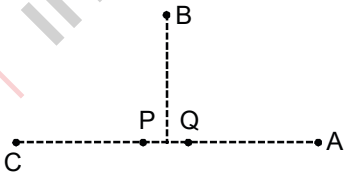
Ans. (4)

Sol. f = 5

$$\gamma = 1 + \frac{2}{f} = 1 + \frac{2}{5} = \frac{7}{5}$$

$$\frac{C_p}{C_v} = \frac{7}{5}$$

18. In the figure below, P and Q are two equally intense coherent sources emitting radiation of wavelength 20 m. The separation between P and Q is 5m and the phase of P is ahead of that of Q by 90°. A, B and C are three distinct point of observation, each equidistant from the midpoint of PQ. The intensities of radiation at A, B, C will be in the ratio :



(1) 0 : 1 : 2 (2) 4 : 1 : 0 (3) 0 : 1 : 4 (4) 2 : 1 : 0

Ans. (4)

Sol. $\Delta P_A = \frac{\pi}{2} \times \frac{20}{2\pi} - 5 = 0$ so $\Delta\phi_A = 0$

$\Delta P_B = \frac{\pi}{2} \times \frac{20}{2\pi} = 5$ so $\Delta\phi_B = \frac{\pi}{2}$

$\Delta P_C = 5 + 5 = 10$ so $\Delta\phi_C = \pi$

$$I = I_0 \cos^2 \frac{\Delta\phi}{2}$$

so $I_A = I_0$

& $I_B = \frac{I_0}{2}$

$I_C = 0$

so $I_A : I_B : I_C = 2 : 1 : 0$

19. A screw gauge has 50 divisions on its circular scale. The circular scale is 4 units ahead of the pitch scale marking, prior to use. Upon one complete rotation of the circular scale, a displacement of 0.5 mm is noticed on the pitch scale. The nature of zero error involved and the lest count of the screw gauge, are respectively :

(1) Positive, 0.1 μm (2) Negative, 2 μm (3) Positive 10 μm (4) Positive, 0.1 mm

Ans. (3)

Sol. Circular scale is 4 unit ahead means screw gage has positive zero error

$= 4 \times 10\ \mu m = 40\ \mu m$

$$\text{Least count} = \frac{0.5}{50} = \frac{1}{100} mm = 10\ \mu m.$$

20. An electron is moving along +x direction with a velocity of 6×10^6 ms^{-1}. It enters a region of uniform electric field of 300 V/cm pointing along +y direction. The magnitude and direction of the magnetic field set up in this region such that the electron keeps moving along the x direction will be :

(1) 5×10^{-3} T, along + z direction (2) 3×10^{-4} T, along +z direction

(3) 3×10^{-4} T, along –z direction (4) 5×10^{-3} T, along –z direction

Ans. (1)

Sol. $qVB = qE$

$$B = \frac{E}{V} = \frac{30000}{6 \times 10^6} = 5 \times 10^3\ T$$

$$\text{Magnetic force } F = q\left(\vec{V} \times \vec{B}\right)$$

So Ans. 5×10^3 T, along +Z direction

SECTION – 2 : (Maximum Marks : 20)

This section contains FIVE (05) questions. The answer to each question is **NUMERICAL VALUE** with two digit integer and decimal upto one digit.

If the numerical value has more than two decimal places truncate/round-off the value upto TWO decimal places.

Full Marks : +4 If ONLY the correct option is chosen.

Zero Marks : 0 In all other cases

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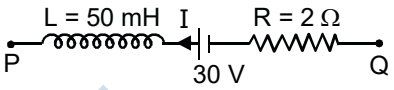
21. A part of a complete circuit is shown in the figure. At some instant, the value of current I is 1A and it is decreasing at a rate of 10^2 A s^{-1}. The value of the potential difference $V_P - V_Q$, (in volts) at that instant is–



Ans. 33

Sol. $V_P - L.\frac{dI}{dt} - 30 + RI = V_Q$

$V_P - V_Q = + 50 \times 10^{-3} \times 10^2 + 30 - 1 \times 2$

$= + 5 + 30 - 2 = 33$

22. Two bodies of the same mass are moving with the same speed, but in different directions in a plane. They have a completely inelastic collision and move together thereafter with a final speed which is half of their initial velocities of the two bodies (in degree) is –

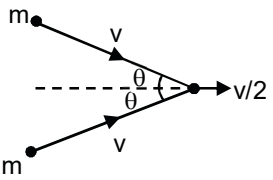
Ans. 120

Sol. Momentum conservation along x direction

$2mv\cos\theta = (2m)\frac{v}{2}$

$\cos\theta = \frac{1}{2}$

$\theta = 60°$



then angle between their initial velocities = 120°

23. The density of a solid metal sphere is diameter. The maximum error in the density of the sphere is $\left(\frac{x}{100}\right)$% . If the relative errors in measuring the mass and the diameter are 6.0% and 1.5% respectively, the value of x is –

Ans. 1050

Sol. $\rho = \frac{m}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$

$\rho = \frac{6m}{\pi d^3}$

taking log

$\ell n\rho = \ell nm - 3\ell nd$

Maximum error by differentiation

$$\frac{d\rho}{\rho} = \frac{dm}{m} + 3\frac{dd}{d}$$

$$\frac{d\rho}{\rho} = 6 + 3 \times 1.5$$

$$= 10.5\% = \frac{1050}{100}\%$$

Ans. x = 1050

24. Suppose that intensity of a laser is $\left(\frac{315}{\pi}\right)$ W / m^2. The rms electric field, in units of V/m associated with this source is close to the nearest integer is – (ε_0 = 8.86 × 10^{-12} C^2 Nm^{-2}; c = 3 × 10^8 ms^{-1})

Ans. 275

Sol. $\frac{1}{2} C \in_0 E_{rms}^2 = \frac{315}{\pi}$

$$E_{rms} = \sqrt{\frac{315 \times 2}{\pi \times 3 \times 10^8 \times 8.86 \times 10^{-12}}}$$

$$= \sqrt{\frac{630}{83.4612 \times 10^{-4}}}$$

$$= \sqrt{7.5484 \times 10^4}$$

= 2.75 × 10^2 = 2.75 × 10^2 = 275

25. Initially a gas of diatomic molecules is contained in a cylinder of volume V_1 at a pressure P_1 and temperature 250 K. Assuming that 25% of the molecules get dissociated causing a change in number of moles. The pressure of the resulting gas at temperature 2000 K, when contained in a volume $2V_1$ is given by P_2. The ratio P_2/P_1 is –

Ans. 5

Sol. $P_1 = P_1$

$v_1 = v_1$

$T_1 = 250$

$n_1 = n_0$

$n_2 = 0.75\ n_0 + 2 \times 0.25\ n_0 = 1.25 n_0$

$v_2 = 2V_1$

$T_2 = 2000$

$P_1V_1 = n_1RT_1$

$P_2V_2 = n_2RT_2$

$P_1\ V_1 = n_0R \times 250$ ...(1)

$P_2(2V_1) = (.25 n_0R\ (2000))$ ...(2)

equation (1) / (2)

$$\frac{P_1}{2P_2} = \frac{250}{1.25 \times 2000}$$

$$\frac{P_1}{P_2} = \frac{1}{5}$$

$$\frac{P_2}{P_1} = 5$$

PART-B : CHEMISTRY

SECTION – 1 : (Maximum Marks : 80)

Single Choice Type

This section contains 20 Single choice questions. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which Only One is correct.

Full Marks : +4 If ONLY the correct option is chosen.

Negative Marks : –1 (minus one) mark will be deducted for indicating incorrect response.

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26. The presence of soluble fluoride ion upto 1 ppm concentration in drinking water, is :

(1) harmful to bones (2) harmful to skin (3) safe for teeth (4) harmful for teeth

Ans. (3)

Sol. NCERT XI-Part-II-Page No.408 (Environmental Chemistry)

Soluble fluoride is often added to drinking water to make concentration upto 1 ppm because F^- ions make the enamel on teeth much harder. But above 2 ppm cause brown mottling of teeth. Cause harmful effect on bones and teeth.

27. Arrange the following solutions in the decreasing order of pOH :

(A) 0.01 M HCl (B) 0.01 M NaOH

(C) 0.01 M CH_3COONa (D) 0.01 M NaCl

(1) (B) > (C) > (D) > (A) (2) (A) > (C) > (D) > (B)

(3) (B) > (D) > (C) > (A) (4) (A) > (D) > (C) > (B)

Ans. (4)

Sol. pH of 0.01 M HCl = 2

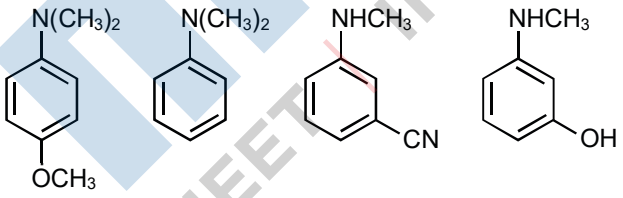
pH of 0.01 M NaOH = 12

pH of 0.01 M CH_3COONa > 7

pH of 0.01 M NaCl = 7

So the order of pH : B > C > D > A

28. The increasing order of pK_b values of the following compounds is :



I II III IV

(1) II < I < III < IV (2) II < IV < III < I (3) I < II < IV < III (4) I < II < III < IV

Ans. (3)

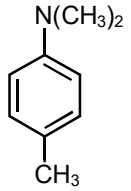
Sol. $PK_b \propto \frac{1}{\text{Basic strength } (K_b)}$

Kb $\propto$ +I, +M effect (e– density)

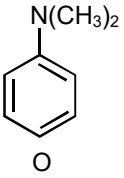
$$K_b \propto \frac{1}{-I,-M \text{ effect}}$$

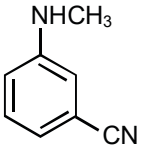
$K_b = 1 > 2 > 4 > 3$

$PK_b = 1 < 2 < 4 < 3$

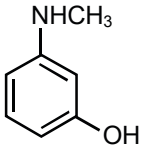
(I) 

+I, +HC

(II) 

(III) 

–I Strong

(IV) 

–I (Weak)

29. Consider the following reactions :

'A' (C_7H_{14}) $\xrightarrow{\text{ozonolysis}}$ 'B' + 'C'

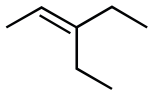
'B' $\xrightarrow[\Delta]{(I_2 + NaOH)}$ yellow ppt

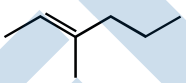
'B' $\xrightarrow[\Delta]{Ag_2O}$ silver mirror

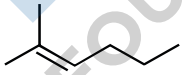
'C' $\xrightarrow[\Delta]{(I_2 + NaOH)}$ no yellow ppt

'C' $\xrightarrow{LiAlH_4}$ 'D' $\xrightarrow[\text{\& Conc. HCl}]{\text{Anhydrous } ZnCl_2}$ gives white turbidity within 5 minutes

'A' is

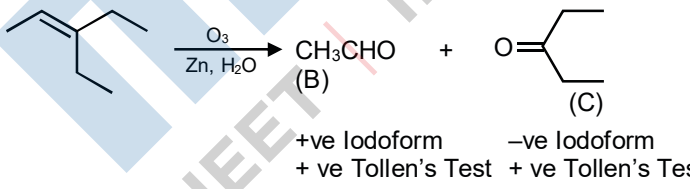
(1) 

(2) 

(3) 

(4) 

Ans. (1)

Sol. 

+ve Iodoform + ve Tollen's Test

–ve Iodoform + ve Tollen's Test

30. Which of the following compounds shows geometrical isomerism ?

(1) 4-methylpent-1-ene

(2) 2-methylpent-2-ene

(3) 4-methylpent-2-ene

(4) 2-methylpent-1-ene

Ans. (3)

Sol. $CH_3 - \underset{\displaystyle CH_3}{\underset{|}{CH}} - CH = CH - CH_3$

Can show G.I. because both doubly bonded carbon have two different groups.

31. The set that contains atomic numbers of only transition elements, is :

(1) 37, 42, 50, 64 (2) 21, 32, 53, 64 (3) 21, 25, 42, 72 (4) 9, 17, 34, 38

Ans. (3)

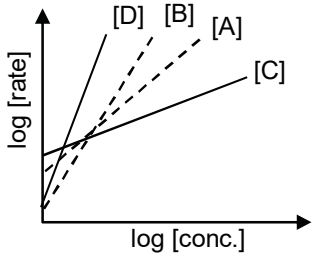
Sol. A transition element is defined as the one which has incompletely filled d orbitals in its ground state or in any one of the oxidation state.

Zn(z = 30), Cd(z = 48), Hg(z = 80) are not transition element.

32. Consider the following reactions

$A \rightarrow P_1$; $B \rightarrow P_2$; $C \rightarrow P_3$; $D \rightarrow P_4$,

The order of the above reactions are a, b, c and d, respectively. The following graph is obtained when log[rate] vs.log[conc.] are plotted :



Among the following, the correct sequence for the order of the reactions is :

(1) d > b > a > c (2) a > b > c > d (3) c > a > b > d (4) d > a > b > c

Ans. (1)

Sol. Rate = $K[Conc]^{order}$

logRate = log K + order log[conc]

so slope of graph is order of reaction.

greater the slope greater is order of reaction

so order of reaction $\Rightarrow$ "A < B < C < D"

33. For the reaction

$$Fe_2N(s)+\frac{3}{2}H_2(g) \rightleftharpoons 2Fe(s)+NH_3(g)$$

(1) $K_c = K_p(RT)$ (2) $K_c = K_p(RT)^{3/2}$ (3) $K_c = K_p(RT)^{1/2}$ (4) $K_c = K_p(RT)^{-1/2}$

Ans. (3)

Sol. $K_c = K_p(RT)^{\Delta ng} = K_c = K_p(RT)^{1/2}$ as $\Delta n_g = 1/2$

34. The variation of equilibrium constant with temperature is given below :

Temperature Equilibrium Constant

$T_1 = 25^oC$ $K_1 = 10$

$T_2 = 100^oC$ $K_2 = 100$

The values of ΔH^o, ΔG^o at T_1 and ΔG^o at T_2 (in kJ mol^{-1}) respectively, are close to

[use R = 8.314 JK–1 mol–1]

(1) 0.64, –7.14 and –5.71 (2) 28.4, –5.71 and –14.29

(3) 28.4, –7.14 and –5.71 (4) 0.64, –5.71 and –14.29

Ans. (2)

Sol. $T_1 = 323$ K $\quad T_2 = 373$ K

$k_1 = 10 \quad k_2 = 100$

$$\log\left(\frac{k_2}{k_1}\right) = \frac{\Delta H}{2.303R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

$$\log\left(\frac{100}{10}\right) = \frac{\Delta H}{2.303 \times 8.314}\left[\frac{1}{298} - \frac{1}{373}\right]$$

$$\log 10 = \frac{\Delta H}{2.303 \times 8.314}\left[\frac{75}{298 \times 373}\right]$$

$$\Delta H = \frac{2.303 \times 8.314 \times 298 \times 373}{75} = 28.4 \text{ KJ}$$

At $T_1 = 25^\circ C = 298$ K, $K_1 = 10$

$\Delta G = -2.303\, RT_1 \log K_1$

$= -2.303 \times 8.314 \times 298 \times \log(10)$

$= -2.303 \times 8.314 \times 298 \times 1$

$= -5.7$ KJ

At $T_2 = 100^\circ C = 373$ K $K_2 = 100$

$\Delta G = -2.303\, RT2 \log K_2$

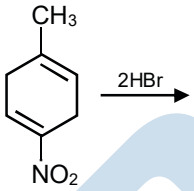
$= -2.303 \times 8.314 \times 373 \times \log(10)^2$

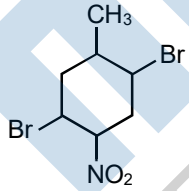
$= -2.303 \times 2 \times 8.314 \times 373 \times 1$

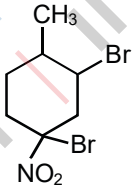
$= -14283.7$ J

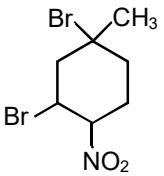
$= -14.29$ KJ

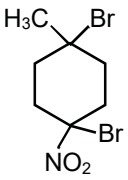
35. The major product of the following reaction is :



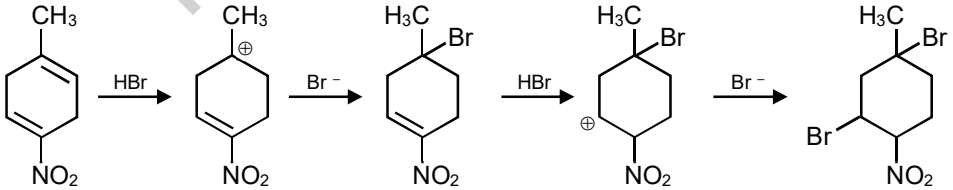
(1) 

(2) 

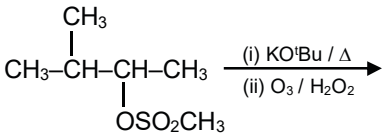
(3) 

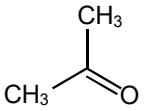
(4) 

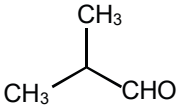
Ans. (3)

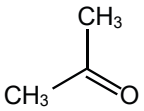
Sol. 

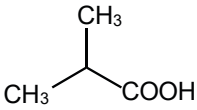
36. The major products of the following reaction are :



(1)  + CH_3CHO

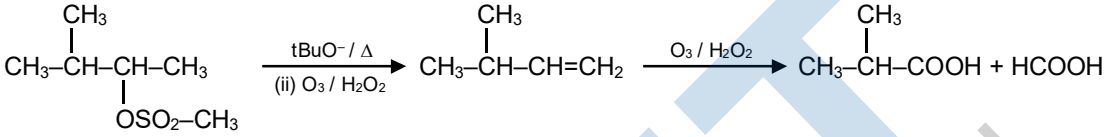
(2)  + HCHO

(3)  + CH_3COOH

(4)  + HCOOH

Ans. (4)

Sol. Due to bulky base at high temperature first reaction is E_2 and produce Hoffman alkene which further show oxidative ozonolysis.



Given in option (C). So answer goes to 3

37. Kraft temperature is the temperature :

(1) above which the aqueous solution of detergents starts boiling.

(2) below which the aqueous solution of detergents starts freezing.

(3) above which the formation of micelles takes place.

(4) below which the formation of micelles takes place.

Ans. (3)

Sol. (i) Micelles formation take place only above a particular temperature called as kraft temperature (Tk)

(ii) Concentration above which micelle formation become appreciable is critical micells concentration.

38. he lanthanoid that does NOT show +4 oxidation state is :

(1) Ce (2) Tb (3) Dy (4) Eu

Ans. (4)

Sol. Cu(z = 63) = $4F^7 6s^2$ Dy(z = 66) = $4F^{10} 6s^2$

Tb(z = 65) = $4F^9 6s^2$ Ce(58) = $4F^1 5d^1 6s^2$

Dy, Tb and Ce show +4 oxidation state while Eu do not show +4 oxidation state.

39. Consider the Assertion and Reason given below.

Assertion (A) : Ethene polymerized in the presence of Ziegler Natta Catalyst at high temperature and pressure is used to make buckets and dustbins.

Reason (R) : High density polymers are closely packed and are chemically inert. Choose the correct answer from the following :

(1) Both (A) and (R) are correct but (R) is not the correct explanation of (A).0

(2) Both (A) and (R) are correct and (R) is the correct explanation of (A).

(3) (A) is correct but (R) is wrong.

(4) (A) and (R) both are wrong.

Ans. (2)

Sol. Based on NCERT-XII-II, Page. No 430 (Polymer)

High density polythene: It is formed when addition polymerisation of ethene takes place in a hydrocarbon solvent in the presence of a catalyst such as triethylaluminium and titanium tetrachloride (Ziegler-Natta catalyst) at a temperature of 333 K to 343 K and under a pressure of 6-7 atmospheres. High density polythene (HDP) thus produced, consists of linear molecules and has a high density due to close packing. It is also chemically inert and more tougher and harder. It is used for manufacturing buckets, dustbins, bottles, pipes, etc.

40. The species that has a spin-only magnetic moment of 5.9 BM, is :

(Td = tetrahedral)

(1) $Ni(CO)_4$ (Td) (2) $[Ni(CN)_4]^{2-}$ (square planar)

(3) $[NiCl_4]^{2-}$ (Td) (4) $[MnBr_4]^{2-}$ (Td)

Ans. (4)

Sol.

(1) $Ni(CO)_4$	$3d^{10}$	0
(2) $[Ni(CN)_4]^{2-}$	$3d^8$	0
(3) $[NiCl_4]^{2-}$	$3d^8$	2
(4) $[MnBr_4]^{2-}$	$3d^5$	5

41. A solution of two components containing n_1 moles of the 1st component and n_2 moles of the 2nd component is prepared. M_1 and M_2 are the molecular weights of component 1 and 2 respectively. If d is the density of the solution in g mL^{-1}, C_2 is the molarity and x_2 is the mole fraction of the 2nd component, then C_2 can be expressed as :

(1) $C_2 = \frac{dx_1}{M_2 + x_2(M_2 - M_1)}$ (2) $C_2 = \frac{1000\, x_1}{M_2 + x_2(M_2 - M_1)}$

(3) $C_2 = \frac{1000\, x_2}{M_2 + x_2(M_2 - M_1)}$ (4) $C_2 = \frac{d\, x_2}{M_2 + x_2(M_2 - M_1)}$

Ans. (3)

Sol. $C_2 = \frac{1000\, x_2}{M_2 + x_2(M_2 - M_1)}$

42. Among the sulphates of alkaline earth metals, the solubilities of $BeSO_4$ and $MgSO_4$ in water, respectively, are :

(1) poor and poor (2) poor and high (3) high and poor (4) high and high

Ans. (4)

Sol.

$BeSO_4$
$MgSO_4$
$CaSO_4$
$SrSO_4$
$BaSO_4$

↓ On moving down Solubility decreases

So, $BeSO_4$ and $MgSO_4$ have high solubility in water.

43. The correct statement with respect to dinitrogen is :

(1) liquid dinitrogen is not used in cryosurgery.

(2) N_2 is paramagnetic in nature

(3) it can combine with dioxygen at 25ºC.

(4) it can be used as an inert diluent for reactive chemicals.

Ans. (4)

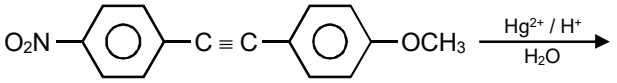
Sol. (1) liquid dinitrogen is used in cryosurgery.

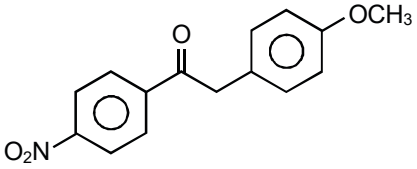
(2) N_2 is diamagnetic in nature.

(3) it can not combine with dioxygen at 25ºC.

(4) is correct

44. The major product obtained from the following reaction is :



(1)


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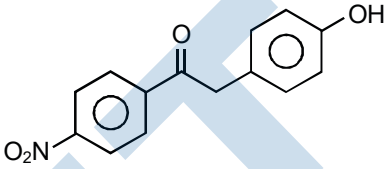

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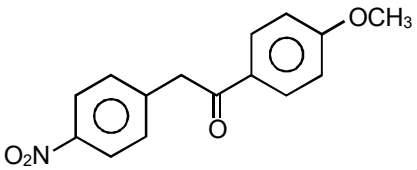
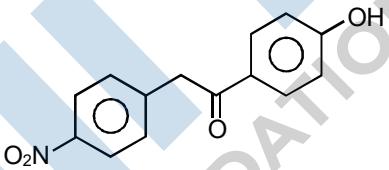
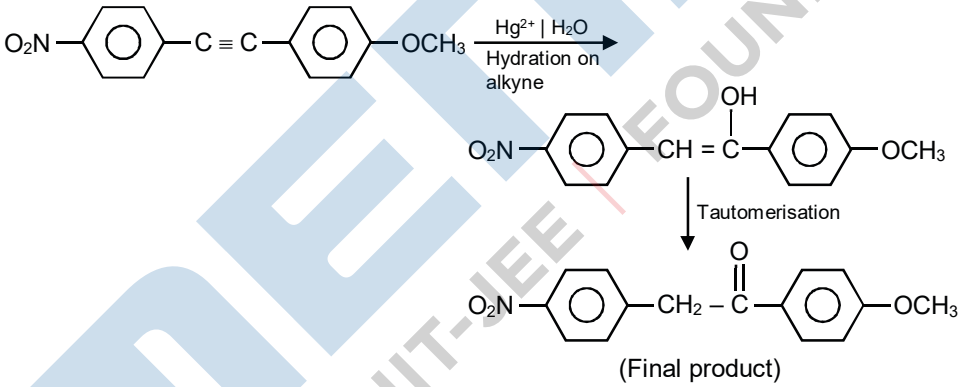


(4)


Ans. (3)

Sol.


(Final product)

45. The INCORRECT statement is :

(1) bronze is an alloy of copper and tin.

(2) brass is an alloy of copper and nickel.

(3) cast iron is used to manufacture wrought iron.

(4) german silver is an alloy of zinc, copper and nickel.

Ans. (2)

Sol.

S. No.	Alloy	Composition
1.	Bronze	Cu (75 – 90%) + Sn (10, 25%)
2.	Brass	Cu (60 – 80%) + Zn (20 – 40%)
3.	German Silver	Cu (50 – 62%) + Zn (17 – 19%) + Ni (21 – 30%)

Cast iron is used to manufacture wrought iron.

SECTION – 2 : (Maximum Marks : 20)

This section contains FIVE (05) questions. The answer to each question is **NUMERICAL VALUE** with two digit integer and decimal upto one digit.

If the numerical value has more than two decimal places truncate/round-off the value upto TWO decimal places.

Full Marks : +4 If ONLY the correct option is chosen.

Zero Marks : 0 In all other cases

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46. the elevation of boiling point of 0.10 m aqueous $CrCl_3.xNH_3$ solution is two times that of 0.05 m aqueous $CaCl_2$ solution. The value of x is ……..

[Assume 100% ionisation of the complex and $CaCl_2$, coordination number of Cr as 6, and that all NH_3 molecules are present inside the coordination sphere]

Ans. (05.00)

Sol. 0.05 m aqueous $CaCl_2$ solution = 0.15 molal solution of non electrolyte

Since the elevation in boiling point is double so the molality of given solution of complex = 0.3 m

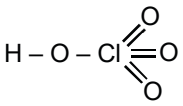
Hence the complex must be producing three ions in the solution, so its formula = $[Cr(NH_3)_5Cl]Cl_2$

so x = 5

47. The number of Cl=O bonds in perchloric acid is, "……."

Ans. (03.00)

Sol. Perchloric acid $\Rightarrow HClO_4$



Total Cl = O bonds = 3

48. Potassium chlorate is prepared by the electrolysis of KCl in basic solution

$6OH^- + Cl^- \rightarrow ClO_3^- + 3H_2O + 6e^-$

If only 60% of the current is utilized in the reaction, the time (rounded to the nearest hour) required to produce 10 g of $KClO_3$ using a current of 2 A is …….

(Given : F = 96,500 C mol^{-}; molar mass of $KClO_3$ = 122 g mol^{-1})

Ans. (11.00)

Sol. $\{2 \times t \times 60 \times 60 / 96500\} \times 0.60 \times 122 \times (1/6) = 10$

So t = 10.98 hours

49. A spherical balloon of radius 3 cm containing helium gas has a pressure of 48×10^{-3} bar. At the same temperature, the pressure, of a spherical balloon of radius 12 cm containing the same amount of gas will be …….. $\times 10^{-6}$ bar.

Ans. (750.00)

Sol. if volume of 3 cm balloon is = V

then volume of 12 cm radius balloon = 64V

So pressure will become 1/64 times = $1/64 \times 48 \times 10^{-3}$ bar = 750×10^{-3} bar

50. In an estimation of bromine by Carius method, 1.6 g of an organic compound gave 1.88 g of AgBr. The mass percentage of bromine in the compound is ……

(Atomic mass, Ag = 108, Br = 80 g mol^{-1})

Ans. (50.00)

Sol. if molar mass of compound is M

1.6/M = 1.88/188 = 0.01

so m = 160 gram/mol

Now %of bromine = 80/160 × 100 % = 50%

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PART-C : MATHEMATICS

SECTION – 1 : (Maximum Marks : 80)

Single Choice Type

This section contains 20 Single choice questions. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which Only One is correct.

Full Marks : +4 If ONLY the correct option is chosen.

Negative Marks : –1 (minus one) mark will be deducted for indicating incorrect response.

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51. Let L_1 be a tangent to the parabola $y^2 = 4(x + 1)$ and L_2 be a tangent to the parabola $y^2 = 8(x + 2)$ such that L_1 and L_2 intersect at right angles. Then L_1 and L_2 meet on the straight line :

(1) $x + 3 = 0$ (2) $2x + 1 = 0$ (3) $x + 2y = 0$ (4) $x + 2 = 0$

Ans. (1)

Sol. Let tangent of $y^2 4(x + 1)$ be

$L_1 : t_1 y = (x + 1) + t_1^2 2$ .....(i)

and tangent of $y^2 = 8(x + 2)$ be

$L_2 : t_2 y = (x + 2) + 2t_2^2$ .....(ii)

$L_1 \perp L_2$

$$\frac{1}{t_1}.\frac{1}{t_2} = -1$$

$t_1 t_2 = -1$

$t_2 \times$ (i) $- t_1 \times$ (ii) $\Rightarrow$ $x + 3 = 0$.

52. If $f(x + y) = f(x) f(y)$ and $\sum_{x=1}^{\infty} f(x) = 2$, $x, y \in N$, where N is the set of all natural numbers, then the value of $\frac{f(4)}{f(2)}$ is :

(1) $\frac{2}{3}$ (2) $\frac{1}{3}$ (3) $\frac{1}{9}$ (4) $\frac{4}{9}$

Ans. (4)

Sol. Let $f(x) = a^x$

$$\sum_{x=1}^{\infty} f(x) = 2$$

$\Rightarrow$ $a + a^2 + a^3 \ldots\ldots\infty$ terms $= 2$

$$\Rightarrow \quad \frac{a}{1-2} = 2 \qquad \Rightarrow \quad a = \frac{2}{3}$$

$$\Rightarrow \quad f(x) = \left(\frac{2}{3}\right)^x$$

Now, $$\frac{f(4)}{f(2)} = \frac{\left(\frac{2}{3}\right)^4}{\left(\frac{2}{3}\right)^2} = \frac{4}{9}.$$

53. If $\sum_{i=1}^{n}(x_i - a) = n$ and $\sum_{i=1}^{n}(x_i - a)^2 = na, (n, a, > 1)$ then the standard deviation of n observations $x_1, x_2, \ldots x_n$ is :

(1) $a - 1$ (2) $\sqrt{n(a-1)}$ (3) $n\sqrt{(a-1)}$ (4) $\sqrt{(a-1)}$

Ans. (4)

Sol. $S.D. = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)}$

$= \sqrt{\frac{\sum (x_i - a)^2}{n} - \left(\frac{\sum (x_i - a)}{n}\right)^2}$

$= \sqrt{\frac{n.a}{n}\left(\frac{n}{n}\right)^2} = \sqrt{a-1}$

54. $A \lim_{x \to 1}\left(\frac{\int_0^{(x-1)^2} t\cos(t^2)dt}{(x-1)\sin(x-1)}\right)$

(1) is equal to 1 (2) does not exist (3) is equal to $\frac{1}{2}$ (4) is equal to $-\frac{1}{2}$

Ans. **(BONUS)**

Sol. $\lim_{x \to 1}\frac{2(x-1).(x-1)^2\cos(x-1)^4}{(x-1)\cos(x-1) + \sin(x-1)}$

$\lim_{x \to 1}\frac{2(x-1)^2\cos(x-1)^4}{\frac{\sin(x-1)}{(x-1)} + \cos(x-1)} = \frac{0}{1+1} = 0.$

55. Which of the following points lies on the locus of the foot of perpendicular drawn upon any tangent to the ellipse, $\frac{x^2}{4} + \frac{y^2}{2} = 1$ from any of its foci?

(1) $(-2.\sqrt{3})$ (2) $(-1.\sqrt{2})$ (3) (1, 2) (4) $(-1.\sqrt{3})$

Ans. (4)

Sol. Foot of perpendicular from focus upon any tangent lies on auxiliary circle $x^2 + y^2 = 4$.

56. Out of 11 consecutive natural number if three numbers are selected at random (without repetition), then the probability that they are in A.P. with positive common difference is :

(1) $\frac{10}{99}$ (2) $\frac{15}{101}$ (3) $\frac{5}{33}$ (4) $\frac{5}{101}$

Ans. (3)

Sol. **Case-1 :** E,O, E,O, E,O, E,O, E,O, E

2b = a + c $\rightarrow$ even $\Rightarrow$ Both a and c should be either even or odd.

$$P = \frac{{}^{6}C_2 + {}^{5}C_2}{{}^{11}C_3} = \frac{5}{33}$$

Case-2 : O, E,O, E,O, E,O, E,O, E,O

$$P = \frac{{}^{5}C_2 + {}^{6}C_2}{{}^{11}C_3} = \frac{5}{33}$$

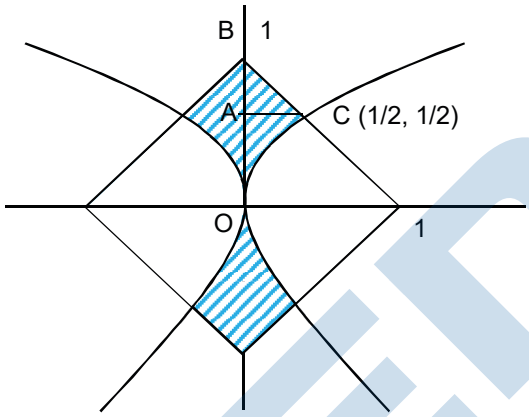
Total probability = $\frac{1}{2}.\frac{5}{33} + \frac{1}{2}.\frac{5}{33} = \frac{5}{33}$..

57. The area (in sq. units) of the region A = $\left\{(x,y) : |x| + |y| \le 1, 2y^2 \ge |x|\right\}$:

(1) $\frac{5}{6}$ (2) $\frac{7}{6}$ (3) $\frac{1}{3}$ (4) $\frac{1}{6}$

Ans. (1)

Sol.



A = ar (OAC) + Δ ABC

$$A = 4.\left[\int_0^{1/2} 2y^2 dy + \frac{1}{2}.\frac{1}{2}.\frac{1}{2}\right] = 4\left[\left(\frac{2}{3}y^3\right)_0^{1/2} + \frac{1}{8}\right]$$

$$= \frac{5}{6}.$$

58. Let m and M be respectively the minimum value values of $\begin{vmatrix} \cos^2 x & 1+\sin^2 x & \sin 2x \\ 1+\cos^2 x & \sin^2 x & \sin 2x \\ \cos^2 x & \sin^2 x & 1+\sin 2x \end{vmatrix}$ Then the ordered pair (m,M) is equal to :

(1) (–3, 3) (2) (1, 3) (3) (–3, –1) (4) (–4, –1)

Ans. (3)

Sol. $C_1 \rightarrow C_1 + C_2$

$$D=\begin{vmatrix}2 & - & -\\ 2 & - & -\\ 1 & - & -\end{vmatrix}$$

$R_1 \to R_1 - 2R_3$ and $R_2 \to R_2 - 2R_3$]

$$D=\begin{vmatrix}0 & \cos^2 x & -(2+\sin 2x)\\ 0 & -\sin^2 x & -(2+\sin 2x)\\ 1 & \sin^2 x & 1+\sin 2x\end{vmatrix}=-2-\sin 2x$$

m = –3, M =–1.

59. Let a, b, c, d and ay non zero distinct real numbers sucjh that $(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)\, p + (b^2 + c^2 + d^2) = 0$, Then :

(1) a, b, c, d are in A.P. (2) a, c, p are in G.P.

(3) a, b, c, d are in G.P. (4) a, c, p, are in A.P.

Ans. (3)

Sol. $(ap - b)^2 + (bp - c)^2 + (cp - d)^2 = 0$

$\Rightarrow$ $ap - b = 0,\ bp - c = 0,\ cp - d = 0$

$\Rightarrow$ $P=\frac{b}{a}=\frac{c}{b}=\frac{d}{c}.$

60. If {p} denotes the fractional part of the number p, then $\left\{\frac{3^{200}}{8}\right\}$, is equal to :

(1) $\frac{1}{8}$ (2) $\frac{3}{8}$ (3) $\frac{7}{8}$ (4) $\frac{5}{8}$

Ans. (1)

Sol. $3^{200} = 9^{100}\,(8 + 1)^{100}$

$= {}^{100}C_0.\ 8^{100} + {}^{100}C_1.\ 8^{99} + \ldots\ldots\ldots + {}^{100}C_{99}.\ 8 + {}^{100}C_{100}$

$\Rightarrow$ $3^{200} = 8\lambda + 1$

$\Rightarrow$ $\frac{3^{200}}{8}=\lambda+\frac{1}{8}.$

$\Rightarrow$ $\left\{\frac{3^{200}}{8}\right\}=\frac{1}{8}.$

61. The values of λ and μ for which the system of linear equations.

$x + y + z = 2$

$x + 2y + 3z = 5$

$x + 3y + \lambda z = \mu$

has infinitely many solutions are respectively :

(1) 5 and 8 (2) 4 and 9 (3) 6 and 8 (4) 5 and 7

Ans. **(1)**

Sol. $D=\begin{vmatrix}1 & 1 & 1\\ 1 & 2 & 3\\ 1 & 3 & \lambda\end{vmatrix}=0 \quad \Rightarrow \quad \lambda=5$

$\Rightarrow \quad D_1=\begin{vmatrix}1 & 1 & 2\\ 1 & 2 & 5\\ 1 & 3 & \lambda\end{vmatrix}=0 \quad \Rightarrow \quad \mu=8.$

62. The region represented by $\{z = x + iy \in C : |z| - Re(z) \le 1\}$ is also given by the inequality

(1) $y^2 \ge 2(x+1)$ (2) $y^2 \le x+\frac{1}{2}$ (3) $y^2 \le \left(x+\frac{1}{2}\right)$ (4) $y^2 \ge x+1$

Ans. (3)

Sol. $\sqrt{x^2+y^2}-x \le 1$

$\Rightarrow \quad \sqrt{x^2+y^2} \le x+1$

$\Rightarrow \quad x^2+y^2 \le x^2+2x+1$

$\Rightarrow \quad y^2 \le 2x+1.$

63. The general solution of the differential equation $\sqrt{1+x^2+y^2+x^2y^2}+xy\frac{dy}{dx}=0$ (where C is constant of integration)

(1) $\sqrt{1+y^2}+\sqrt{1+x^2}=\frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right)+C$

(2) $\sqrt{1+y^2}+\sqrt{1+x^2}=\frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}+1}\right)+C$

(3) $\sqrt{1+y^2}-\sqrt{1+x^2}=\frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right)+C$

(4) $\sqrt{1+y^2}-\sqrt{1+x^2}=\frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right)+C$

Ans. **(2)**

Sol. $\int \frac{y}{\sqrt{1+y^2}}dy=\int -\frac{\sqrt{1+x^2}}{x}dx$

$\int \frac{2y}{2\sqrt{1+y^2}}dy=\int -\frac{1+x^2}{x\sqrt{1+x^2}}dx$

$\Rightarrow \quad \sqrt{1+y^2}=-\int \frac{x}{\sqrt{1+y^2}}dx-\int \frac{x}{x^2\sqrt{1+x^2}}dx$

$\Rightarrow \quad \sqrt{1+y^2}=-\sqrt{1+x^2}-\int \frac{t}{(t^2-1)t}dt$

$$\Rightarrow \quad \sqrt{1+y^2}+\sqrt{1+x^2} = -\int \frac{1}{t^2-1}dt$$

$$= \quad \frac{1}{2(1)}\ell n\left(\frac{t+1}{t-1}\right)+C$$

$$\Rightarrow \quad \sqrt{1+y^2}+\sqrt{1+x^2} = \frac{1}{2}\ell n\left[\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right]+C.$$

64. The position of moving car at time is given by $f(t) = at^2 + bt + c, > 0$, where a, b and c are real numbers greater than 1. Then the average speed of the car over the time interval $[t_1, t_2]$ is attained at the point :

(1) $(t_1 + t_3)/2$ (2) $(t_2 - t_1)/2$ (3) $2a(t_1 + t_2) + b$ (4) $a(t_2 - t_1)+b$

Ans. **(1)**

Sol. $$f(t)=V_{avg}=\frac{f(t_2)-f(t_1)}{t_2-t_1}=\frac{a(t_2^2-t_1^2)+b(t_2-t_1)}{t_2-t_1}$$

$\Rightarrow \quad a(t_1+t_2) + b = 2at + b$

$$t=\frac{t_1+t_2}{2}.$$

65. The negation of the Boolean expression $p \vee (\sim p \wedge q)$ is equivalent to :

(1) $p \wedge \sim q$ (2) $\sim p \vee q$ (3) $\sim p \wedge \sim q$ (4) $\sim p \vee \sim q$

Ans. (3)

Sol. Given statement is $p \vee (q \wedge \sim p)$

$\therefore$ Negation $\sim (p \vee (q \wedge \sim p))$

$= (\sim p \wedge \sim q) \vee (\sim q \wedge p)$

$= \sim p \wedge \sim q$

66. Two families with three members each and one family with four members are to be seated in a row. In how many ways can they be seated so that the same family members are not separated ?

(1) 2! 3! 4! (2) 3! $(4!)^3$ (3) (3!)2.(4!) (4) $(3!)^3$.(4!)

Ans. **(4)**

Sol. $3! \times 3! \times 3! \times 4! = (3!)^3 \times 4!$.

67. The shortest distance between the lines $\frac{x-1}{0}=\frac{y+1}{-1}=\frac{z}{1}$ and $x + y + z + 1 = 0$, $2x - y + z + 3 = 0$ is

(1) $\frac{1}{2}$ (2) 1 (3) $\frac{1}{\sqrt{3}}$ (4) $\frac{1}{\sqrt{2}}$

Ans. **(3)**

Sol. Plane through line of intersection is $x+ y + z +1 + \lambda(2x - y + z + 3) = 0$

It should be parallel to given line

$\Rightarrow \quad 0(1 + 2\lambda) - 1(1 - \lambda) + 1((1 + \lambda) = 0$

$\Rightarrow \quad \lambda = 0$

Plane $\rightarrow x + y + z + 1 = 0$

S.D. = Perpendicular distance of (1,–1,0) from this plane

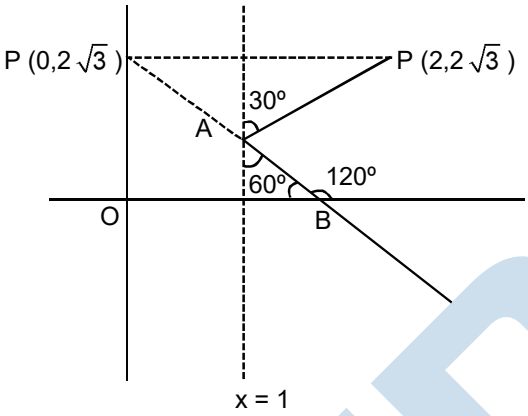
$$= \frac{|1-1+0+1|}{\sqrt{\sqrt{1^2+1^2+1^2}}} = \frac{1}{\sqrt{3}}$$

68. A ray of light coming from the point $\left(2, 2\sqrt{3}\right)$ is incident at an angle 30º on the line x = 1 at the point A. The ray gets reflected on the line x = 1 and meets x-axis at the point B. Then, the line AB passes through the point ;

(1) $\left(4, -\frac{\sqrt{3}}{2}\right)$ (2) $\left(3, -\sqrt{3}\right)$ (3) $\left(4, -\sqrt{3}\right)$ (4) $\left(3, -\frac{1}{\sqrt{3}}\right)$

Ans. **(2)**

Sol.



Equation of P'B $\rightarrow y - 2\sqrt{3} = \tan 120º . (x - 0)$

$$\Rightarrow \quad \sqrt{3}x + y = 2\sqrt{3}$$

69. If $I_1 = \int_0^1 \left(1-x^{50}\right)^{100} dx$ and $I_2 = \int_0^1 \left(1-x^{50}\right)^{101} dx$ $I_2 = \alpha I_1$ then α equals to :

(1) $\frac{5049}{5050}$ (2) $\frac{5051}{5050}$ (3) $\frac{5050}{5051}$ (4) $\frac{5050}{5049}$

Ans. **(3)**

Sol. $$\alpha = \frac{\int_0^1 \left(1-x^{50}\right)^{100} dx}{\int_0^1 \left(1-x^{50}\right)^{101} dx} = \frac{I_2}{I_1}$$

$$I_2 = \int_0^1 \left(1-x^{50}\right)\left(1-x^{50}\right)^{100} dx$$

$$I_2 = I_1 - \int_0^1 x.x^{49}\left(1-x^{50}\right)^{100} dx$$

$$I_2 = I_1 - \left[\frac{-x\left(1-x^{50}\right)^{101}}{5050}\right]_0^1 - \int_0^1 \frac{\left(1-x^{50}\right)^{100}}{5050}dx$$

$$I_2 = I_1 - \frac{I_2}{5050}$$

$$\Rightarrow \qquad \alpha = \frac{I_2}{I_1} - \frac{5050}{5051}.$$

70. If α and β be two roots of the equation $x^2 - 64x + 256 = 0$. Then the value of $\left(\frac{\alpha^3}{\beta^5}\right)^{\frac{1}{8}} + \left(\frac{\beta^3}{\alpha^5}\right)^{\frac{1}{8}}$ is :

(1) 2 (2) 4 (3) 3 (4) 1

Ans. (1)

Sol. $\alpha + \beta = 64$, $\alpha\beta = 256$

$$\text{Now} = \left(\frac{\alpha^3}{\beta^5}\right)^{\frac{1}{8}} + \left(\frac{\beta^3}{\alpha^5}\right)^{\frac{1}{8}}$$

$$= \frac{\alpha^{\frac{3}{8}+\frac{5}{8}} + \beta^{\frac{3}{8}+\frac{5}{8}}}{(\alpha\beta)^{5/8}} = \frac{\alpha+\beta}{(\alpha\beta)^{5/8}}$$

$$= \frac{64}{(256)^{5/8}} = \frac{64}{32} = 2$$

SECTION – 2 : (Maximum Marks : 20)

This section contains FIVE (05) questions. The answer to each question is **NUMERICAL VALUE** with two digit integer and decimal upto one digit.

If the numerical value has more than two decimal places truncate/round-off the value upto TWO decimal places.

Full Marks : +4 If ONLY the correct option is chosen.

Zero Marks : 0 In all other cases

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71. If $\vec{a}$ and $\vec{b}$ are unit vectors, then the greatest value of $\sqrt{3}|\vec{a}+\vec{b}|+|\vec{a}-\vec{b}|$ is

Ans. (4)

Sol. Let $\vec{a} \wedge \vec{b} = \alpha$

$$\sqrt{3}|\vec{a}+\vec{b}|+|\vec{a}-\vec{b}| = \sqrt{3}\sqrt{2+2\cos\alpha}+\sqrt{2-2\cos\alpha}$$

$$= \sqrt{3}\sqrt{2\times 2\cos^2\frac{\alpha}{2}}+\sqrt{2\times 2\sin^2\frac{\alpha}{2}}$$

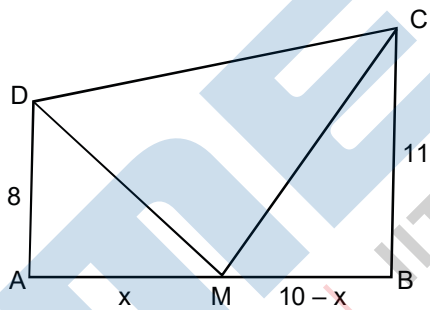
$$= 2\left(\sqrt{3}\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}\right)$$

Maximum value = 2(2) = 4.

72. Let AD and BC be two vertical poles at A and B respectively on a horizontal ground. If AD = 8 m, BC = 11 m and AB = 10 m; then the distance (in meters) of a point M on AB from the point A such that $MD_2 + MC_2$ is minimums is ____.

Ans. (5)

Sol.



$MD_2 + MC_2 = 64 + x^2 + 121 + (10 - x)^2$

$\Rightarrow \quad D = 285 - 20x + 2x^2$

$$\frac{dD}{dx} = 0 \Rightarrow \quad x = 5.$$

73. The angle of elevation of the top of a hill from a point on the horizontal plane passing through the foot of the hill is found to be 45°. After waling a distance of 80 meters towards the top, up a slope inclined at angle of 30° to the horizontal plane the angle of elevation of the top of the hill becomes 75°. Then the height of the hill (in meters) is _____.

Ans. (80)

Sol. $x = 80\cos 30° = 40\sqrt{3}$

$y = 80 \sin 30° = 40$

$\tan 45° = 1$

$\Rightarrow \quad x + z = h$

$\Rightarrow \quad 40\sqrt{3} + z = h \quad$ .....(i)

$\tan 75° = \dfrac{h-y}{z}$

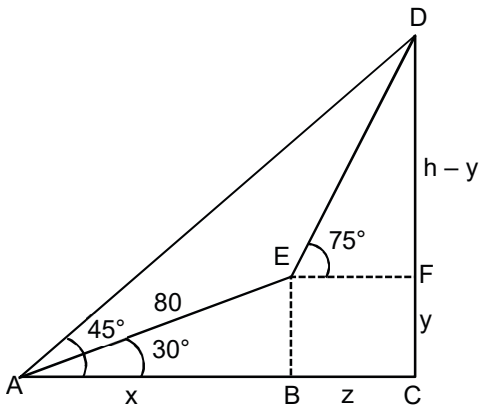
$\Rightarrow \quad 2+\sqrt{3} = \dfrac{h-40}{z}$

$\Rightarrow \quad z = \dfrac{h-40}{2+\sqrt{3}}$

$\Rightarrow \quad h - 40\sqrt{3} = \dfrac{h-40}{2+\sqrt{3}}$

$(1+\sqrt{3})h = 80(1+\sqrt{3})$

$h = 80.$



74. Set A has m elements and Set B has n elements. If the total number of subsets of A is 112 more than the total number of subsets of B, then the value of m-n is _____.

Ans. (28)

Sol. $2^m = 2^n + 112$

$2^m - 2^n = 112$

$\Rightarrow \quad 2^n \cdot (2^{m-n} - 1) = 2^4 \cdot (2^3 - 1)$

$\Rightarrow \quad n = 4 , m = 7.$

75. Let $f : R \to R$ be defined as

$$f(x) = \begin{cases} x^5 \sin\left(\dfrac{1}{x}\right) + 5x^2, & x < 0 \\ 0 & x = 0 \\ x^5 \cos\left(\dfrac{1}{x}\right) + \lambda x^2, & \end{cases}$$

The value of λ for which f''(0) exists, is_____.

Ans. (05)

Sol. $$f(x) = \begin{cases} 5x^4.\sin(1/x) - x^3\cos(1/x) + 10x & x < 0 \\ 0 & x = 0 \\ 5x^4.\cos(1/x) + x^3\sin(1/x) + 2\lambda x & x > 0 \end{cases}$$

$$f(x) = \begin{cases} 20x^3.\sin(1/x) - 5x^2\cos(1/x) - 3x^2\cos(1/x) - x\sin(1/x) + 10 & x < 0 \\ 0 & x = 0 \\ 20x^{30}.\cos(1/x) + 5x^2\sin(1/x) + 3x^2\sin(1/x) - x\cos(1/x) + 2\lambda & x > 0 \end{cases}$$

$f'(0^+) = f'(0^-)$

$\Rightarrow \quad 2\lambda = 10 \quad \Rightarrow \quad \lambda = 5.$